

Problems for Recitation 11

1. Give a description of the equivalence classes associated with each of the following equivalence relations.
 - (a) Integers x and y are equivalent if $x \equiv y \pmod{3}$.
 - (b) Real numbers x and y are equivalent if $\lceil x \rceil = \lceil y \rceil$, where $\lceil z \rceil$ denotes the smallest integer greater than or equal to z .

2. Show that neither of the following relations is an equivalence relation by identifying a missing property (reflexivity, symmetry, or transitivity).
 - (a) The “divides” relation on the positive integers.
 - (b) The “implies” relation on propositional formulas.

3. Here is prerequisite information for some MIT courses:

18.01 \rightarrow 6.042	18.01 \rightarrow 18.02
18.01 \rightarrow 18.03	6.046 \rightarrow 6.840
8.01 \rightarrow 8.02	6.01 \rightarrow 6.034
6.042 \rightarrow 6.046	18.03, 8.02 \rightarrow 6.02
6.01, 6.02 \rightarrow 6.003	6.01, 6.02 \rightarrow 6.004
6.004 \rightarrow 6.033	6.033 \rightarrow 6.857

- (a) Draw a Hasse diagram for the corresponding partially-ordered set. (A **Hasse diagram** is a way of representing a poset (A, \preceq) as a directed acyclic graph. The vertices are the element of A , and there is generally an edge $u \rightarrow v$ if $u \preceq v$. However, self-loops and edges implied by transitivity are omitted.) You'll need this diagram for all the subsequent problem parts, so be neat!
- (b) Identify a largest chain. (A **chain** in a poset (S, \preceq) is a subset $C \subseteq S$ such that for all $x, y \in C$, either $x \preceq y$ or $y \preceq x$.)
- (c) Suppose that you want to take all the courses. What is the minimum number of terms required to graduate, if you can take as many courses as you want per term?
- (d) Identify a largest **antichain**. (An **antichain** in a poset (S, \preceq) is a subset $A \subseteq S$ such that for all $x, y \in A$ with $x \neq y$, neither $x \preceq y$ nor $y \preceq x$.)

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